Reply to "Comment on 'Ward identities for transport of classical waves in disordered media'"

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We respond to the Comment on our previous work by Barabanenkov and Ozrin. We note that their result for the simple case of a scalar wave system, which seems correct by itself, is not necessarily in conflict with our results, which are derived on grounds of very general and rigorous considerations. In the case of the scalar wave, the combination of their result and ours implies an even simpler Ward identity relating the self-energy and the Bethe-Salpeter kernel.

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The derivation of a Ward identity for the simple case of the scalar wave given by Barabanenov and Ozrin [1] is interesting, and their result seems correct. But, their criticism of our paper [2] is grossly misplaced. We have full confidence in our result. What is happening, we believe, is that both results are correct, implying an even simpler and stronger relationship between the self-energy and the Bethe-Salpeter kernel, at least in this simple case of the scalar wave. We outline our views with the following sequence of comments.

The formalism of quantum field theory, which we used in our original paper [2], is a natural choice for the study of classical wave transport in disordered media. First, classical fields are the limiting cases of their respective quantum fields. Second, ensemble averaging over statistical configurations for the description of the disordered nature of the media, under rather general assumptions, results in algorithms identical to those of the Feynman diagrams developed in quantum field theory. One should not lose sight of the fact that some freely adopted results in the literature, like the Bethe-Salpeter equation, derive their validity on the basis of such algorithms. For a detailed enunciation, there is the recent book by Rammer [3].

We have carefully reexamined the crucial equations responsible for the results given in our paper [2]. They are essentially the Ward-Takahashi identity for energy conservation, our original Eq. (28),

$$\partial_x^\mu \Gamma_\mu(\eta|x|\zeta) = G^{(e)-1}(\eta-x)\frac{\partial}{\partial t_x}\delta^4(x-\zeta) + G^{(e)-1}(x-\zeta)\frac{\partial}{\partial t_x}\delta^4(\eta-x). \quad (1)$$

and the integral equation for the energy-density vertex function, our original Eq. (30),

$$\Gamma_\mu(\eta|x|\zeta) = \Gamma_\mu^{(0)}(\eta|x|\zeta) - \int d^4y\, d^4y'\, d^4z\, d^4z'\, G^{(e)}(z'-z) \times \Gamma_\mu(z|x|y) G^{(e)}(y-y') K(\eta\zeta; y'z'), \quad (2)$$

where K is the Bethe-Salpeter kernel, and the vertex function Γ_μ for the energy density $T_{\mu 0}$ is defined by

$$\langle 0|T\{T_{\mu 0}(x)\phi(y)\phi(x)\}|0\rangle^{\mathrm{av}} = \int d^4\eta\, d^4\zeta \frac{1}{i} G^{(e)}(y-\eta)\Gamma_\mu(\eta|x|\zeta)\frac{1}{i}G^{(e)}(\zeta-z). \quad (3)$$

None of these two equations is called into question. The structure of the Ward-Takahasi identity (1) cannot possibly be wrong; it carries the unmistakable signature of the linear time derivative, appropriate for the energy operator to play the role of generating time displacement. The validity of the integral equation (2), which Barabanenov and Ozrin allege to be invalid, is actually on the same footing as the Bethe-Salpeter equation, both resulting from the same kind of Feynman-diagram analysis. In fact, the structure of integral equation (2) was borrowed from the corresponding integral equation for the current-density vertex function in quantum electrodynamics [4], as we explicitly stated in our original paper. To the extent that the Bethe-Salpeter equation is valid for the system considered, so is the integral equation (2).

In another publication [5], similar considerations were applied to electronic systems with random static impurities, for which both energy and charge are conserved, and which possess symmetry property under spin rotation. The resulting set of Ward identities, all involving the self-energy and the Bethe-Salpeter kernel, imply fairly stringent conditions on the relationship between the self-energy Σ and the Bethe-Salpeter kernel K. The end result is a simple relationship between the self-energy and the Bethe-Salpeter kernel. In the present case of scalar wave, a similar situation seems to present itself. The result obtained by Barabanenov and Ozrin [1].

$$\omega_2^2 M_{p+}(\omega_1) - \omega_1^2 M_{p^-}(\omega_2) = \int_{p'} K_{pp'}(q;\omega_1,\omega_2)[\omega_1^2 G_{p'_+}(\omega_1) - \omega_2^2 G_{p'_-}(\omega_2)] \quad (4)$$

and the result obtained by us [2],

$$q_0\Sigma(q+p)-(q_0+p_0)\Sigma(q) = \int \frac{d^3\vec{\alpha}}{(2\pi)^3}[q_0 G^{(e)}(q+\vec{\alpha})-(q_0+p_0) \times G^{(e)}(q+p+\vec{\alpha})]K(q+p,-q,\vec{\alpha}), \quad (5)$$

though being different, need not be mutually exclusive; they are both valid. Taken together, they imply the following simpler yet stronger relation:

$$\Sigma(q)=\int \frac{d^3\vec{\alpha}}{(2\pi)^3}G^{(e)}(q+p+\vec{\alpha})K(q,q+p,-\vec{\alpha}). \quad (6)$$

We have checked and verified this relation in terms of low-order Feynman diagrams; the situation is very much like what appeared in the case of the electronic systems. We have also searched in the present scalar-wave case for another conservation law other than energy conservation that might be responsible for the simplified relationship, but have not yet succeeded in finding one.

The formalism we used has been easily generalized to more complicated classical-wave systems in disordered media, such as the elastic waves and the electromagnetic waves [2]. It would be of interest to investigate how the method of Barabanenkov and Ozrin can be applied to such systems, or even to an alternative version of the scalar-wave equation.

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